Progression in Calculations

Addition

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| Objective and Strategies | | Concrete | Pictorial | Abstract |
| Combining two parts to make a whole: part- whole model | | Use cubes to add two numbers together as a group or in a bar. | C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].png  Image result for part whole model addition  Use pictures to add two numbers together as a group or in a bar.  8  1  C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].png  C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngImage result for part whole modelC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].png | 4 + 3 = 7  10= 6 + 4  5  3  Use the part-part whole diagram as shown above to move into the abstract. |
| Starting at the bigger number and counting on | Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  Ten frame activities to help become fluent.  Moving on to spotting and applying  known mental recall facts to solve calculations. | | 12 + 5 = 17      Start at the larger number on the number line and count on in ones or in one jump to find the answer.  Encourage and discuss the use of mental recall facts, to find the answer at all times. | 5 + 12 = 17  Place the larger number in your head and count on the smaller number to find your answer.  Moving on to spotting and  applying known mental  recall facts to solve  calculations.  Eg:  2 + 5 = 7  12 + 5 = 17  OR  5 = 2 + 3  12 + 3 = 15  15 + 2 = 17 |
| Regrouping to make 10 | 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.  Spotting and applying known  mental recall facts to 10, to solve calculations. | | Use pictures or a number line. Regroup or partition the smaller number to make 10. | 7 + 4= 11  If I am at seven, how many more do I need to make 10. How many more do I add on now?  Partition the 4 into 3 and 1  7 + 3 = 10  10 + 1 = 11 |
| Adding three single digits | 7 + 2 + 3 = 12  Put the 7 and the 3 together to make 10. Add on the 2.      Exploring other methods involving  regrouping eg:  9 + 3 + 2 = 14  Put 3 and 2 together to make 5.  Group with 9 using previous  strategy. | | Find bonds to 10; rearrange calculation if necessary. Renaming one number to make 10.    7 + 4 + 9 = 20    Find bonds to 10 within calculations.  e.g. partition 4 into 3 and 1, then apply 7 + 3 and 1 + 9 which becomes 10 + 10 = 20. | Combine the two numbers that make 10 and then add on the remainder.  Find bonds to 10 within  calculations.  Look for patterns and use partitioning and known additive facts to make calculation simpler. |
| Partitioning with no regrouping/recombining | 24 + 13 = 37  Use the Base 10 blocks first before moving onto place value counters. | | After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  Jotting alongside use of apparatus. |  |
| Partitioning- regrouping/renaming | Other resources can be used, such as base ten or place value counters. | | Jotting alongside use of apparatus.  This could be used alongside a similar structure to the part-part-whole model above. | 28 = 20 + 8  39 = 30 + 9  50 + 17 = 67 |
| Column method – no regrouping | 24 + 15  Att together the ones first then add the tens.  Represent this using Base 10 and place value counters. | | After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  T O |  |
| Column method – regrouping/renaming | Make both numbers on a place value grid.    Add up the ones and exchange 10 ones for one 10.    Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.  As children move on to decimals, money and decimal place value counters can be used to support learning. | | Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. | Start by partitioning the numbers before moving on to clearly show the exchange through the line.  **Note that carried numbers are recorded to the left and on the line. We have agreed not to use 0 as a place holder for ‘missing’ digits.**  127 + 498 = 625    As the children move on, introduce decimals with the same number of decimal places and then different. Money can be used here. |



Subtraction

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| Objective and Strategies | Concrete | Pictorial | Abstract |
| Taking away ones | Use physical objects, counters, cubes etc to show how objects can be taken away.    Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. | Cross out drawn objects to show what has been taken away. | 6 – 2 = 4  15 – 3 = 12 |
|
| Find the difference | Compare amounts and objects to find the difference.    Image result for two towers of cubes  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference | Count on to find the difference.  http://image.slidesharecdn.com/intro-to-sm-1220840292402057-8/95/intro-to-singapore-math-13-728.jpg?cb=1345557040  Draw bars to find  the difference between two numbers. | Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.  23-15= |
| Part Part Whole Model | Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part?  10 - 6 = | Use a pictorial representation of objects to show the part part whole model. | 10  5  Move to using numbers within the part whole model. |
| Make 10 | 14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. | Start at the smaller number, the 13. How many more to get to the next ten? Seven. How many more to get to 27? Now add the two parts together to find the difference between 27 and 13. | 16 – 8=  How many do we take off to reach the next 10?  How many do we have left to take off? |
| Column method without regrouping | Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first. | Draw the Base 10 or place value counters alongside the written calculation to help to show working. | [https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9GcS1ohiHkzn0cS0nvwRP-5EyK0TDGl_A1tbsAl0XjNPBssTas4YVeQ](http://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0CAcQjRxqFQoTCPyKt_H6h8kCFUNEFAodiFAGCA&url=http://huppiemama.com/teaching-subtraction-using-manipulatives/&bvm=bv.106923889,d.d2s&psig=AFQjCNEr_xOQu7fhwvMOMFTIen6kpdc03g&ust=1447317198959935)This will lead to a clear written column subtraction. |
| Column method with regrouping | Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.  Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.    Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.    Now I can take away eight tens and complete my subtraction    Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount. | Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping.  Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.  Show children how the concrete method links to the written method alongside your working out. Cross out your numbers when exchanging and show where we write our new amount. | Less confident children may be shown the expanded method for subtraction (above).  Moving forward, the children use a more compact method.    This will lead to an understanding of subtracting any number including decimals. |



Multiplication

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| Objective and Strategies | Concrete | Pictorial | Abstract |
| Make, find and see equal groups. | Use different objects to make, find and see equal groups e.g. take four plates and put an equal group on each plate. | Representing and finding equal groups using pictorial methods.    Which activity has groups of 3? Or groups of 4? | Link pictorial representations for  abstract examples using names  and numbers without  mentioning of operational  symbol (x) e.g. |
| Doubling | Use practical activities to show how to double a number. | Draw pictures to show how to double a number. | Partition a number and then double each part before recombining it back together. |
| Counting in multiples | Count in multiples supported by concrete objects in equal groups.    e.g. solving 6 x 4 by drawing / making  6 groups of 4 and skip counting to find  the total. | Use a number line or pictures to continue support in counting in multiples. | Count in multiples of a number aloud.  Write sequences with multiples of numbers.  2, 4, 6, 8, 10  5, 10, 15, 20, 25 , 30 |
| Repeated addition | Use different objects to add equal groups. | Children can record this as a bar model:      The dotted line shows the unknown quantity. Children could then replace the question mark with the number 40. | Write addition sentences to describe objects and pictures.  Children can then record this  onto an empty number line:  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  0 5 10 15 20 25 30 35 40  8 x 5 = 40  2 + 2 + 2 = 3 x 2 |
| Arrays- showing commutative multiplication | Create arrays using counters/ cubes to show multiplication sentences.  http://www.australiancurriculumlessons.com.au/wp-content/uploads/2013/05/arrays-multiplication-division-lesson.jpg | Draw arrays in different rotations to find **commutative** multiplication sentences.    http://mathcentral.uregina.ca/QQ/database/QQ.02.06/maro1.1.gif  Link arrays to area of rectangles.  Associated facts | Use an array to write multiplication sentences and reinforce repeated addition. |
| **Grid** | Show the link with arrays to first introduce the grid method.    4 rows of 10  4 rows of 3  Move on to using Base 10 to move  towards a more compact method.    Move on to place value counters to  show how we are finding groups of a  number. We are multiplying by 4 so we need 4 rows.    Fill each row with 126. | Children can represent the work they have done with place value counters in a way that they understand.  They can draw the counters, using  colours to show different amounts  or just use circles in the different  columns to show their thinking as  shown below. | Start with multiplying by one  digit numbers and showing the  clear addition alongside the grid.    Moving forward, multiply by a 2  digit number showing the  different rows within the grid  method |
| Column multiplication | https://primarysite-prod.s3.amazonaws.com/0c4eb252d34643748228179a3d582154_1x1.jpegChildren can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer, followed by the tens, which they note below.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. | Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (7).JPG  *C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (5).JPG* | Start with long multiplication, reminding the children about lining up their numbers clearly in columns. |

Division

It was agreed that children need to be exposed to images that support grouping and sharing models of division from the beginning.

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| Objective and Strategies | Concrete | Pictorial | Abstract |
| Sharing objects into equal groups | The counters are sharded (individually moved, one by one) equally into two groups into this example.    I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities.    8 ÷ 2 = 4 | Share 9 buns between three people.  9 ÷ 3 = 3 |
| Division as grouping | Divide quantities into equal groups.  Use cubes, counters, objects or place value counters to aid understanding. | Use a number line to show jumps in groups. The number of jumps equals the number of groups.    Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. | 28 ÷ 7 = 4  Divide 28 into groups of 7. How many groups of 7 will you have?  This can be represented on a number line if needed. |
| Division within arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15 | Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating four linking number sentences.  7 x 4 = 28  4 x 7 = 28  28 ÷ 7 = 4  28 ÷ 4 = 7 |
| Division with a remainder | 14 ÷ 3 =  Divide objects between groups and see how much is left over  Image result for counters | Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. | http://amsi.org.au/teacher_modules/G7/G7_qt2%202.pngComplete written divisions and show the remainder using r.  Challenge children to convert remainders into fractions e.g.  29 ÷ 4 = 7 ¼  Remainder becomes numerator and group amount becomes  denominator.  These can then be converted into  decimals e.g.  29 ÷ 4 = 7 ¼  ¼ = 0.25  29 ÷ 4 = 7.25 |
| Short division | Use place value counters to divide using the bus stop method alongside  42 ÷ 3=  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.    We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14. | http://www.studyzone.org/testprep/math4/d/division2.gifStudents can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. | Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. |